

Evaluate the following limits.

SCORE: ____ / 11 PTS

Your answer should be a number, ∞ , $-\infty$ or DNE (only if the first three answers do not apply).

[a] $\lim_{x \rightarrow 0} \frac{x \sin x}{e^{3x} - 3e^x + 2}$ $\frac{0 \cdot 0}{1 - 3 + 2} \rightarrow \frac{0}{0}$

$= \lim_{x \rightarrow 0} \frac{\sin x + x \cos x}{3e^{3x} - 3e^x} \quad (2) \quad \frac{0+0}{3-3} \rightarrow \frac{0}{0}$

$= \lim_{x \rightarrow 0} \frac{\cos x + \cos x - x \sin x}{9e^{3x} - 3e^x} \quad (2)$

$= \frac{1+1-0}{9-3} = \frac{2}{6} = \frac{1}{3} \quad (1)$

[b] $\lim_{x \rightarrow 0^+} (1 - \sin x)^{\frac{1}{x}}$

$= \lim_{x \rightarrow 0^+} e^{\ln(1 - \sin x)^{\frac{1}{x}}}$

$= e^{\lim_{x \rightarrow 0^+} \frac{\ln(1 - \sin x)}{x}}$

$= e^{-1} \quad (1\frac{1}{2})$

$\lim_{x \rightarrow 0^+} \frac{\ln(1 - \sin x)}{x} \quad (2) \quad \frac{0}{0}$

$= \lim_{x \rightarrow 0^+} \frac{\frac{1}{1 - \sin x} \cdot -\cos x}{1} \quad (1\frac{1}{2})$

$= \frac{1}{1-0} \cdot -1 = -1 \quad (1)$

Using complete sentences and proper mathematical notation, state the formal definition of "local maximum". SCORE: ____ / 3 PTS

f HAS A LOCAL MAXIMUM AT c

GRADED BY ME

IF $f(x) \leq f(c)$ FOR ALL x IN AN OPEN INTERVAL
AROUND c

Find the absolute extrema of $f(x) = 3x^{\frac{2}{3}}(x-10)$ on the interval $[-1, 8]$.

SCORE: ____ / 7 PTS

$$\begin{aligned} f'(x) &= 2x^{-\frac{1}{3}}(x-10) + 3x^{\frac{2}{3}} \\ &= x^{-\frac{1}{3}}(2x-20+3x) \\ &= \boxed{x^{-\frac{1}{3}}(5x-20)} \quad (1) \end{aligned}$$

$$\boxed{f' \text{ DNE @ } x=0} \quad (1)$$

$$\boxed{f' = 0 \text{ @ } x=4} \quad (1)$$

x	$f(x)$
-1	$3(1)(-9) = \underline{-27}$ (1/2)
0	$\underline{0}$ (1/2) (1/2)
4	$3 \cdot 4^{\frac{2}{3}}(-6) = -18^3 \sqrt[3]{16} = \underline{-36^3 \sqrt[3]{2}}$ (1/2)
8	$3(4)(-2) = \underline{-24}$ (1/2)

ABSOLUTE MAX @ $x=0$ (1)

MIN @ $x=4$ (1)

Consider the following three cases:

SCORE: ____ / 9 PTS

Case 1: $f(x) = (x-1)^{\frac{2}{3}}$ on the interval $[-7, 9]$ CONT, NOT DIFF $f' = \frac{2}{3}(x-1)^{-\frac{1}{3}}$ DNE @ $x=1$

Case 2: $g(x) = \frac{4}{4-x}$ on the interval $[0, 3]$ CONT, DIFF $g' = -4(4-x)^{-2}(-1) = \frac{4}{(4-x)^2}$

Case 3: $h(x) = \frac{6}{x^2-1}$ on the interval $[-2, 2]$ DISCONT @ $x = \pm 1$

- [a] Does Rolle's Theorem apply to any of the three cases? **If no, write "N/A" (no other work required).**
If yes, list all cases to which it applies, and list all the conditions of Rolle's Theorem which are satisfied.

N/A ①

- [b] Does the Extreme Value Theorem apply to any of the three cases? **If no, write "N/A" (no other work required).**
If yes, list all cases to which it applies, and list all the conditions of the Extreme Value Theorem which are satisfied.

CASES 1, 2: ①

FUNCTIONS ARE CONTINUOUS ON CLOSED + BOUNDED INTERVALS
① ②

- [c] Does the Mean Value Theorem apply to any of the three cases? **If no, write "N/A" (no other work required).**
If yes, list all cases to which it applies, and for each of those cases, list all the conditions of the Mean Value Theorem which are satisfied, and find the value of c guaranteed by the Mean Value Theorem.

CASE 2 ① g IS CONT + DIFF ② ③

$$g'(c) = \frac{g(3) - g(0)}{3 - 0} = \frac{4 - 1}{3} = 1$$

$$\textcircled{2} \left[\frac{4}{(4-c)^2} = 1 \right] \rightarrow 4 = (4-c)^2 \rightarrow 4-c = \pm 2 \rightarrow c = 2, 6 \quad \textcircled{1} \left[c=2 \right] \in (0, 3)$$